1. Governing equations and methods

We consider an incompressible infinite Prandtl number fluid in a rectangular domain. Under the extended Boussinesq approximation, the three unknowns, velocity \( \mathbf{u} \), the pressure \( P \) and the temperature \( T \) are determined by solving the conservation of momentum (Stokes equation), mass, and energy equations [cf. Schubert et al., 2001]:

\[
\nabla \cdot (\eta \nabla \mathbf{u}) + \nabla P = \rho \mathbf{g} \quad (1)
\]

\[
\nabla \cdot \mathbf{u} = 0 \quad (2)
\]

\[
\rho C_p (\mathbf{T} + \mathbf{u} \nabla T) = \nabla \cdot (k \nabla T) + f \quad (3)
\]

where the operator \( \nabla^n \) is the symmetric gradient, namely \( 1/2(\nabla^T + \nabla) \), \( \eta \) is the viscosity, \( \rho \) the density, \( \mathbf{g} \) the gravitational acceleration vector, \( C_p \) the isobaric heat capacity, \( k \) the thermal conductivity \( [P - T]-dependent according to Hofmeister, 1999] , and \( f \) a volumetric heat source term which is the sum of three components: i) an adiabatic heating term \( = \rho \mathbf{T} \alpha \rho \mathbf{u} \), ii) a heating term \( = \sigma_{ij} e_{ij} \), and iii) a constant radiogenic heat production term \( Q \). Note that in the “classic” extended Boussinesq approximation, the density in eq. (3) is replaced according to

\[
\rho_0(0, T) = \rho_0(0, T) \quad (a T^2 + \beta T^3 + \gamma T^4) \quad (5)
\]

The pressure effect on the CTE is described by the Anderson-Grünen parameter exactly as in Afonso et al. [2005].

There is general agreement that two main creep mechanisms are likely responsible for most of the deformation in the mantle: diffusion creep (Herring-Nabarro and Coble creep) and dislocation creep [cf. Ranalli, 1995; Karato, 2008]. Deformation caused by dislocation creep is evidenced in xenoliths and peridotic massifs, and indirectly inferred in the shallow upper mantle from seismic anisotropy studies (see Nettles and Dziewonski [2008] for a recent review). On the other hand, diffusion creep may be dominant over dislocation creep at depths > 250-300 km, where stresses are low and pressure effects become dominant [Ranalli, 1995; Karato, 2008].

When using the creep parameters of Karato and Wu [1995], the appropriate flow law has the form

\[
\dot{\epsilon} = A(\sigma/\mu)^n (b/d)^m \exp \left(-\frac{E^* + PV}{RT}\right) \quad (6)
\]

where \( d \) is the average grain-size, \( \sigma \) the differential stress, \( A \) the pre-exponential factor, \( \mu \) the shear modulus, \( b \) the length of the Burgers vector, \( n \) the stress exponent, \( m \) the grain-size exponent, \( E \) the activation energy, \( V \) the activation volume, and \( R \) the gas constant. When creep parameters are taken from Hirth and Kohlstedt [2003], the appropriate flow law to use is (note that we ignore melt-present effects)

\[
\dot{\epsilon} = A(\sigma/\mu)^n \exp \left(-\frac{E^* + PV}{RT}\right) \quad (7)
\]

where \( C_{OH} \) is water content and \( r \) its exponent. All other parameters have the same meaning as in eq. (6). Note that the activation energy \( E^* \) and volume \( V^* \) already include the \( T \) and \( P \) dependence of OH dissolution in olivine [cf. Hirth and Kohlstedt, 2003; Karato, 2008]. All relevant parameters used to solve eqs. (6) and (7) are listed in Table 1.

Applying the Levy-von Mises formalism to purely viscous fluids [Ranalli, 1995; Karato, 2008] allows defining the viscosity as

\[
\eta = \frac{1}{2} (\sigma_{ij} r)^{1-n} A^{-1} = \frac{1}{2} (\varepsilon_{ij} r)^{1-n} A^{-1/2} \quad (8)
\]

where \( \sigma_{ij} r \) is the second invariant of the deviatoric stress tensor \( = (\varepsilon_{ij} r)^{1/2} \), \( \varepsilon_{ij} \) the second invariant of the strain rate tensor \( = (\varepsilon_{ij} r)^{1/2} \) and \( A \) is a factor derived from the empirical Eqs. (6) and (7) and the type of experiment (for details see Chapters 4 of Ranalli [1995].)
and 3 of Karato [2008]). Note that in the case of diffusion creep (n = 1), the viscosity becomes independent of stress. Eqs. (6-8) permit to obtain explicit expressions for η in terms of T, P and d, which is in turn used to solve eq. (1). We iterate for this coupling (Picard method) at each time step until the relative error in the pressure and velocity fields is < 1%. Since diffusion and dislocation creep are thought to act simultaneously in the mantle, two different viscosities ηdiff and ηdisl are computed separately and then combined into an effective viscosity ηeff. The latter is computed as the harmonic mean of ηdiff and ηdisl:

\[ \frac{1}{\eta_{\text{eff}}} = \left( \frac{1}{\eta_{\text{diff}}} + \frac{1}{\eta_{\text{disl}}} \right)^{-1}. \]  

(9)

This expression is truncated if the resulting viscosity is either greater or lower than two imposed cutoff values (10^{18} to 10^{24} Pa s). Creep parameters for "wet" and "dry" rheologies are taken from Karato and Wu [1993] and Hirth and Kohlstedt [2003], and analyzed separately (see paper). Although grain-size may change due to grain growth and dynamic recrystallization processes, its dependence on mantle flow conditions is poorly known. Cpx therefore we consider only constant grain sizes. For average grain sizes d ≥ 3.5 mm, dislocation creep represents an important component of the effective viscosity at depths < 200 km. For smaller values of d, diffusion creep becomes dominant. In this context, we note that synthetic seismological models of oceanic mantle suggest d > 3.5 mm [Afonso et al., 2008]. All simulations shown in this work are carried out with a constant d = 5 mm (Table 1). Fig. 1 depicts a representative thermal field from one of our simulations.

2. Melting models

The amount of melt extracted from the upwelling mantle beneath MORs (F), as well as its depth distribution, play major roles in defining the composition (including H2O content) of oceanic plates [e.g. Afonso et al., 2007]. In this context, the most important parameter controlling the depth dependence of F is the so-called melt productivity function γ = dF/4P [e.g. Asimow et al., 1999]. Although it is a common practice in geodynamic modeling to assume γ constant throughout the melting regime, more realistic thermodynamic simulations show that its behavior is characterized by at least two (or three, if melting continues beyond Cpx exhaustion) well defined paths with nearly constant γ [Asimow et al., 1999, 2001, 2004]. These studies indicate that for the case of "normal" MORs, γ has a low near-solidus productivity at pressures ≥ 1.0 - 1.5 GPa, which rapidly increases to a nearly constant value of ~ 3 % GPa at lower pressures. If clinopyroxene is totally consumed, there is a shallow third segment in which γ decreases dramatically to values < 1.5 %GPa [Asimow et al., 2001, 2004]. Since the shape of the F(P) function reflects the changing behaviour of γ, the former cone also exhibits two (three if Cpx is exhausted) segments of different, but nearly constant, slope [e.g. Asimow et al., 1999, 2001]. As Afonso et al. [2007] and Afonso et al. [2008] demonstrated, the general trend of F(P) can be adequately described, for the purposes of geodynamic modeling, as two (three) linear functions that intercept at the depth at which γ changes its slope. Fig. 2 shows the melting models used in this work, together with a summary of the results presented by Asimow et al. [1999], Asimow et al. [2001], Katz et al. [2003], and Asimow et al. [2004] for both dry and hydrous melting. Our melting models differ only in the initial pressure of melting Pb. Although our parameterization is very crude, all models produce realistic amounts of melts (leading to crustal thicknesses between ~ 6 - 8.5 km) and agree qualitatively with more complex models.

Once a particular melting model is assumed, the H2O content of the solid residue can be modeled as function of melt extraction making use of the H2O bulk partition coefficient (Dw). It is generally believed that melting and melt extraction occur simultaneously in the melting regime [cf. Langmuir et al., 1992]. Therefore, melting can be approximated as a series of infinitesimally small batch melts that are continuously extracted from the system in successive steps. The composition of the residue of each step is then used as the reference for the next step. This method is widely used in both major and trace element modeling and is commonly referred to as nearfractional melting [Asimow and Longhi, 2004, and references therein]. Here we apply this model to estimate the H2O content of the residue assuming ~ 1 % melt porosity (i.e. melt retention), although variations of this value have little effect on the final result. We adopt a constant Dw = 0.008, according to various experimental estimations [Aubaud et al., 2004; Hirschmann, 2006; Hauri et al., 2006, and references therein]. Although it is acknowledged that Dw may vary with H2O fugacity, which in turn would imply a strong dependence on pressure and H2O content [Asimow et al., 2004], the limited available experiments do not evidence this behaviour [cf. Hirschmann, 2006]. Moreover, thermodynamic simulations constrained by experimental results suggest that Dw do not vary significantly unless the H2O content drops to values < 1 % of its original value [Hauri et al., 2006]. Fig. 3 shows the H2O content of the residual peridote as a function of F for different initial concentrations representative of the MORB source. For this range of initial concentrations, the residue becomes virtually dry after 2 - 4 % of partial melting. We therefore vary the parameter ξ (refer to the paper) in our calculations within this range.

References


Table 1. Sets of rheological parameters used in eqs. (6) and (7). Note that not all of these parameters were experimentally constrained at the time of publication (e.g. activation volumes and water content sensitivity; see Mei & Kohlstedt [2000a; 2000b] and Karato & Jung [2003] for more recent estimations). However, here we keep the original values of the tabulated sets for consistency.

Karato & Wu (1993)

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<tr>
<td>$n$</td>
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<td>3.5</td>
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<tr>
<td>$E$</td>
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<tr>
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Hirth & Kohlstedt (2003)

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<td>$V^*$</td>
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<td>14$^\dagger$</td>
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<tr>
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<tr>
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<tr>
<td>$m$</td>
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<tr>
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<tr>
<td>$r$</td>
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</tr>
</tbody>
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$^\dagger$ These are assumed values within the range tabulated in Hirth & Kohlstedt [2003].
Figure 1. Typical thermal structure resulting from our SSC simulations. Temperature scale and contours in Kelvin.

Figure 2. Melt fraction vs pressure for the different melting models used in this work (colour lines). The two dotted and two dash-dotted curves are isentropic melting models for fractional and batch (dry) melting models at different \( P_0 \), respectively [from Asimow et al., 1999, 2001]. The results from Katz et al. [2003] and Asimow et al. [2004] for deep hydrous melting are also shown for comparison. The simple approach used by Katz et al. [2003] to model melt productivity is likely to result in an overestimation of \( F \) for low degrees of melting [Katz et al., 2003; Asimow et al., 1999].
Figure 3. Curves showing the H$_2$O content (in wt ppm) of the residual solid mantle as function of extracted melt (F) for different initial H$_2$O contents. A constant $D_w = 0.008$ and a near-fractional melting model were assumed in the calculations. The rectangle denotes the range of water contents in olivine for which it can be considered effectively dry ($< 10$ ppm). For a MORB source with initial H$_2$O between 50 - 300 ppm, the residue becomes effectively dry after only 2 - 4% melt extraction.